```
> restart;
> ?Physics
> with(Physics):
> Setup(mathematicalnotation = true)
```

$$[mathematicalnotation = true] \tag{1}$$

```
> ds2 := -exp(2*Phi(r))*dt^2+exp(2*Lambda(r))*dr^2+r^2*dtheta^2+
  r^2*sin(theta)^2*dphi^2;
```

$$ds2 := -e^{2\Phi(r)}\, dt^2 + e^{2\Lambda(r)}\, dr^2 + r^2\, d\theta^2 + r^2 \sin(\theta)^2\, d\varphi^2 \tag{2}$$

```
> Setup(coordinates = spherical, metric = ds2)
```

** Partial match of 'coordinates' against keyword 'coordinatesystems'*

Default differentiation variables for d_, D_ and dAlembertian are: $\{X = (r, \theta, \phi, t)\}$

Systems of spacetime Coordinates are: $\{X = (r, \theta, \phi, t)\}$

$$\left[coordinatesystems = \{X\},\ metric = \left\{(1,1) = e^{2\Lambda(r)},\ (2,2) = r^2,\ (3,3) = r^2 \sin(\theta)^2,\ (4,4) = -e^{2\Phi(r)}\right\}\right] \tag{3}$$

```
> Christoffel[alpha, beta, gamma, nonzero]
```

$$\Gamma_{\alpha,\beta,\gamma} = \left\{(1,1,1) = \left(\frac{d}{dr}\Lambda(r)\right) e^{2\Lambda(r)},\ (1,2,2) = -r,\ (1,3,3) = -r \sin(\theta)^2,\ (1,4,4) = \left(\frac{d}{dr}\Phi(r)\right) e^{2\Phi(r)},\ (2,1,2) = r,\ (2,2,1) = r,\ (2,3,3) = -r^2 \sin(\theta)\cos(\theta),\ (3,1,3) = r\sin(\theta)^2,\ (3,2,3) = r^2 \sin(\theta)\cos(\theta),\ (3,3,1) = r\sin(\theta)^2,\ (3,3,2) = r^2\sin(\theta)\cos(\theta),\ (4,1,4) = -\left(\frac{d}{dr}\Phi(r)\right) e^{2\Phi(r)},\ (4,4,1) = -\left(\frac{d}{dr}\Phi(r)\right) e^{2\Phi(r)}\right\} \tag{4}$$

```
> Christoffel[~alpha, beta, gamma, nonzero]
```

$$\Gamma^{\alpha}{}_{\beta,\gamma} = \left\{(1,1,1) = \frac{d}{dr}\Lambda(r),\ (1,2,2) = -e^{-2\Lambda(r)}\, r,\ (1,3,3) = -e^{-2\Lambda(r)}\, r\sin(\theta)^2,\ (1,4,4) = e^{-2\Lambda(r)+2\Phi(r)}\left(\frac{d}{dr}\Phi(r)\right),\ (2,1,2) = \frac{1}{r},\ (2,2,1) = \frac{1}{r},\ (2,3,3) = -\sin(\theta)\cos(\theta),\ (3,1,3) = \frac{1}{r},\ (3,2,3) = \frac{\cos(\theta)}{\sin(\theta)},\ (3,3,1) = \frac{1}{r},\ (3,3,2) = \frac{\cos(\theta)}{\sin(\theta)},\ (4,1,4) = \frac{d}{dr}\Phi(r),\ (4,4,1) = \frac{d}{dr}\Phi(r)\right\} \tag{5}$$

```
> Riemann[~alpha, beta, gamma, delta, nonzero]
```

$$R^{\alpha}{}_{\beta,\gamma,\delta} = \left\{ (1,2,1,2) = \left(\frac{d}{dr}\Lambda(r)\right) e^{-2\Lambda(r)}\, r,\ (1,2,2,1) = -\left(\frac{d}{dr}\Lambda(r)\right) e^{-2\Lambda(r)}\, r,\ (1,3,1,3) = \left(\frac{d}{dr}\Lambda(r)\right) e^{-2\Lambda(r)}\, r\sin(\theta)^2,\ (1,3,3,1) = -\left(\frac{d}{dr}\Lambda(r)\right) e^{-2\Lambda(r)}\, r\sin(\theta)^2,\ (1,4,1,4) = e^{-2\Lambda(r)+2\Phi(r)}\left(\frac{d^2}{dr^2}\Phi(r) - \left(\frac{d}{dr}\Phi(r)\right)\left(\frac{d}{dr}\Lambda(r)\right) + \left(\frac{d}{dr}\Phi(r)\right)^2\right),\ (1,4,4,1) = -e^{-2\Lambda(r)+2\Phi(r)}\left(\frac{d^2}{dr^2}\Phi(r) - \left(\frac{d}{dr}\Phi(r)\right)\left(\frac{d}{dr}\Lambda(r)\right) + \left(\frac{d}{dr}\Phi(r)\right)^2\right),\ (2,1,1,2) = -\frac{\frac{d}{dr}\Lambda(r)}{r},\ (2,1,2,1) = \frac{\frac{d}{dr}\Lambda(r)}{r},\ (2,3,2,3) = -\sin(\theta)^2\left(-1 + e^{-2\Lambda(r)}\right),\ (2,3,3,2) = \sin(\theta)^2\left(-1 + e^{-2\Lambda(r)}\right),\ (2,4,2,4) = \frac{e^{-2\Lambda(r)+2\Phi(r)}\left(\frac{d}{dr}\Phi(r)\right)}{r},\ (2,4,4,2) = -\frac{e^{-2\Lambda(r)+2\Phi(r)}\left(\frac{d}{dr}\Phi(r)\right)}{r},\ (3,1,1,3) = -\frac{\frac{d}{dr}\Lambda(r)}{r},\ (3,1,3,1) = \frac{\frac{d}{dr}\Lambda(r)}{r},\ (3,2,2,3) = -1 + e^{-2\Lambda(r)},\ (3,2,3,2) = 1 - e^{-2\Lambda(r)},\ (3,4,3,4) = \frac{e^{-2\Lambda(r)+2\Phi(r)}\left(\frac{d}{dr}\Phi(r)\right)}{r},\ (3,4,4,3) = -\frac{e^{-2\Lambda(r)+2\Phi(r)}\left(\frac{d}{dr}\Phi(r)\right)}{r},\ (4,1,1,4) = \frac{d^2}{dr^2}\Phi(r) - \left(\frac{d}{dr}\Phi(r)\right)\left(\frac{d}{dr}\Lambda(r)\right) + \left(\frac{d}{dr}\Phi(r)\right)^2,\ (4,1,4,1) = -\frac{d^2}{dr^2}\Phi(r) + \left(\frac{d}{dr}\Phi(r)\right)\left(\frac{d}{dr}\Lambda(r)\right) - \left(\frac{d}{dr}\Phi(r)\right)^2,\ (4,2,2,4) = \left(\frac{d}{dr}\Phi(r)\right) e^{-2\Lambda(r)}\, r,\ (4,2,4,2) = -\left(\frac{d}{dr}\Phi(r)\right) e^{-2\Lambda(r)}\, r,\ (4,3,3,4) = \left(\frac{d}{dr}\Phi(r)\right) e^{-2\Lambda(r)}\, r\sin(\theta)^2,\ (4,3,4,3) = -\left(\frac{d}{dr}\Phi(r)\right) e^{-2\Lambda(r)}\, r\sin(\theta)^2 \right\} \qquad (6)$$

```
> Ricci[beta, delta, nonzero]
```

$$R_{\beta,\delta} = \left\{ (1,1) = \frac{-\left(\frac{d}{dr}\Phi(r)\right)^2 r + \left(\frac{d}{dr}\Phi(r)\right) r \left(\frac{d}{dr}\Lambda(r)\right) - \left(\frac{d^2}{dr^2}\Phi(r)\right) r + 2\frac{d}{dr}\Lambda(r)}{r}, (2,2) = 1 + \left(-\left(\frac{d}{dr}\Phi(r)\right) r + \left(\frac{d}{dr}\Lambda(r)\right) r - 1\right) e^{-2\Lambda(r)}, (3,3) = -\sin(\theta)^2 \left(\left(\frac{d}{dr}\Phi(r)\right) e^{-2\Lambda(r)} r - \left(\frac{d}{dr}\Lambda(r)\right) e^{-2\Lambda(r)} r + e^{-2\Lambda(r)} - 1\right), (4,4) = \frac{1}{r}\left(\left(\left(\frac{d^2}{dr^2}\Phi(r)\right) r + \left(\frac{d}{dr}\Phi(r)\right)\left(\left(\frac{d}{dr}\Phi(r)\right) r - \left(\frac{d}{dr}\Lambda(r)\right) r + 2\right)\right) e^{-2\Lambda(r)+2\Phi(r)}\right)\right\} \quad \textbf{(7)}$$

```
> Ricci[~beta, delta, nonzero]
```

$$R^{\beta}{}_{\delta} = \left\{ (1,1) = -\frac{1}{r}\left(e^{-2\Lambda(r)}\left(\left(\frac{d}{dr}\Phi(r)\right)^2 r - \left(\frac{d}{dr}\Phi(r)\right) r \left(\frac{d}{dr}\Lambda(r)\right) + \left(\frac{d^2}{dr^2}\Phi(r)\right) r - 2\frac{d}{dr}\Lambda(r)\right)\right), (2,2) = \frac{1 + \left(-\left(\frac{d}{dr}\Phi(r)\right) r + \left(\frac{d}{dr}\Lambda(r)\right) r - 1\right) e^{-2\Lambda(r)}}{r^2}, (3,3) = \frac{1 + \left(-\left(\frac{d}{dr}\Phi(r)\right) r + \left(\frac{d}{dr}\Lambda(r)\right) r - 1\right) e^{-2\Lambda(r)}}{r^2}, (4,4) = -\frac{e^{-2\Lambda(r)}\left(\left(\frac{d^2}{dr^2}\Phi(r)\right) r + \left(\frac{d}{dr}\Phi(r)\right)\left(\left(\frac{d}{dr}\Phi(r)\right) r - \left(\frac{d}{dr}\Lambda(r)\right) r + 2\right)\right)}{r}\right\} \quad \textbf{(8)}$$

```
> R:=Ricci[~1, 1]+Ricci[~2, 2]+Ricci[~3, 3]+Ricci[~4, 4]
```

$$R := \frac{1}{r}\left(e^{-2\Lambda(r)}\left(-\left(\frac{d}{dr}\Phi(r)\right)^2 r + \left(\frac{d}{dr}\Phi(r)\right) r \left(\frac{d}{dr}\Lambda(r)\right) - \left(\frac{d^2}{dr^2}\Phi(r)\right) r + 2\frac{d}{dr}\Lambda(r)\right)\right) + \frac{2\left(-\left(\frac{d}{dr}\Phi(r)\right) e^{-2\Lambda(r)} r + \left(\frac{d}{dr}\Lambda(r)\right) e^{-2\Lambda(r)} r - e^{-2\Lambda(r)} + 1\right)}{r^2} + \frac{1}{r}\left(e^{-2\Lambda(r)}\left(\left(\frac{d}{dr}\Phi(r)\right) r \left(\frac{d}{dr}\Lambda(r)\right) - \left(\frac{d}{dr}\Phi(r)\right)^2 r - \left(\frac{d^2}{dr^2}\Phi(r)\right) r - 2\frac{d}{dr}\Phi(r)\right)\right) \tag{9}$$

```
> R:=simplify(R);
```

$$R := \frac{1}{r^2}\left(2 + \left(-2r^2\left(\frac{d^2}{dr^2}\Phi(r)\right) - 2r^2\left(\frac{d}{dr}\Phi(r)\right)^2 + \left(2r^2\left(\frac{d}{dr}\Lambda(r)\right) - 4r\right)\left(\frac{d}{dr}\Phi(r)\right) + 4\left(\frac{d}{dr}\Lambda(r)\right) r - 2\right) e^{-2\Lambda(r)}\right) \tag{10}$$

```
> Ein:=Einstein[beta, delta, nonzero]
```

$$Ein := G_{\beta,\delta} = \left\{ (1,1) = \frac{-e^{2\Lambda(r)} + 2\left(\frac{d}{dr}\Phi(r)\right) r + 1}{r^2}, (2,2) = r\left(\left(\frac{d^2}{dr^2}\Phi(r)\right) r + \left(\frac{d}{dr}\Phi(r) - \frac{d}{dr}\Lambda(r)\right)\left(\left(\frac{d}{dr}\Phi(r)\right) r + 1\right)\right) e^{-2\Lambda(r)}, (3,3) = r\sin(\theta)^2 e^{-2\Lambda(r)}\left(\left(\frac{d^2}{dr^2}\Phi(r)\right) r + \left(\frac{d}{dr}\Phi(r) - \frac{d}{dr}\Lambda(r)\right)\left(\left(\frac{d}{dr}\Phi(r)\right) r + 1\right)\right), (4,4) = \frac{e^{-2\Lambda(r) + 2\Phi(r)}\left(e^{2\Lambda(r)} + 2\left(\frac{d}{dr}\Lambda(r)\right) r - 1\right)}{r^2} \right\} \tag{11}$$